

Inversion of a gradient of nonlinear mechanical properties by mixing of ultrasound surface waves

Period:	10/2026 - 10/2029
Disciplines:	Acoustics, Ultrasounds, Non destructive evaluation of materials
Location:	Université Gustave Eiffel, Nantes Campus
Laboratory:	Geophysics and Non Destructive Evaluation (GéoEND)
Doctorate school:	Sciences Pour l'Ingénieur (SPI)
Funding:	Université Gustave Eiffel
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Key words: Gradients of properties, Inverse problem, Nondestructive testing, Nonlinear acoustics, Rayleigh wave

Background and purpose: Characterizing a gradient of mechanical properties at the surface of a material is a preliminary step to numerous applications in nondestructive evaluation. Such a gradient can indeed inform on the presence and ingress of a damage, on the quality of a surface treatment, or more generally on a factor that influences the remaining lifespan. This thesis focuses more specifically on the context of early detection of deterioration within the first centimeters of concrete, whose purpose is to protect steel rebars from corrosion. An important example which is currently not well detected non-destructively is the carbonation process.

Compared to linear ultrasounds, nonlinear ultrasounds generally offer increased sensitivity to micro-structure changes and has emerged as a promising alternative. In particular, recent works have shown that the nonlinear wave mixing effect that generates a P wave from two surface waves [1] has an interesting potential for a heterogeneous and dissipative material such as concrete [2]. Similar to other linear or non-linear effects [3], varying the frequency of primary waves enables probing the material through the depth. To date, a proof of concept has been given for the detection of a surface damage [2], however without performing a real characterization of the depth profile. The objective of this thesis is to develop a method of resolution of the inverse problem based on this wave mixing effect, in view of quantifying a variation of nonlinear mechanical properties through the depth of the material.

Objectives: The work of this thesis will primarily consist in numerical modelling, using the Python language and the Fenicsx finite elements library.

1. Modelling numerically the wave mixing phenomenon between two Rayleigh waves in a half space with a gradient of nonlinear properties and homogeneous or heterogeneous linear properties. Nonlinearity will be modelled following Murnaghan's hyperelasticity.

Parameters will be chosen to represent wave propagation in concrete, with a wavelength of a few centimeters.

2. Formulate an inverse problem to deduce a depth profile of nonlinear properties from surface data. The target is on the first 3 to 5 centimeters of concrete. The study can be extended to other materials and scales, as for instance to the first 0.2 to 1mm of steel.
3. Based on synthetic data, quantify the resolution of this new method. Study its robustness to degraded conditions (noise, limited frequency coverage of the input data, heterogeneity or attenuation of the material, etc) and find strategies to mitigate them.
4. Apply the inversion method to existing or new experimental data.

References:

- [1] S. Gartsev, P. Zuo, M. Rjelka, A. Mayer & B. Koehler, Nonlinear interaction of Rayleigh waves in isotropic materials: Numerical and experimental investigation, *Ultrasonics* (2022), 122, 106664
- [2] M. Fengal, Evaluation of gradients of nonlinear mechanical properties using ultrasounds: Application to thermally damaged concrete, thèse de doctorat (2025), Univ Gustave Eiffel
- [3] P. Mora & M. Spies, Rayleigh wave harmonic generation in materials with depth-dependent non-linear properties, *JASA* (2018), vol. 143 (5), p. 2678-2684

Expected profile:

- Master degree in acoustics or mechanics.
- Solid background in wave propagation.
- Solid background in numerical modelling.
- Python programming.
- English language.

Resources:

You will have access to:

- modeling codes based on Finite Elements, to develop the method and test it on synthetic data.
- computation resources, for scientific computing.
- existing experimental datasets, to apply the method on real data.
- homemade experimental equipment, to obtain new datasets.
- international conferences, to present the results to peers.